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Most-attractive-channel study
for bulk and brane fermions in warped space

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Abstract

The idea of the self-breaking of the standard model gauge symmetry is applied to a gauge theory in a warped space. We systematically examine the gauge couplings of bulk and brane fermions. The constraint on the masses of bulk fermions is found under the conditions that the ordinary four-dimensional massless gluon is not condensed and that color-triplet scalar bound states are not formed. For bulk fermions with zero modes for the left- and right-handed components, the mass parameters are required to be at least $c \gtrsim 1/2$ and $c \lesssim -1/2$, respectively. Then possible vacuum expectation values arise only from weak-doublet scalar bound states which trigger electroweak symmetry breaking.

1   Introduction

In Nature, there are multiple elementary particles with only the values of their masses different from each other. It is unknown what the origin of the masses is. In the standard model of particle physics, fermions and gauge bosons acquire their masses by symmetry breaking through the Higgs mechanism. While the standard model has passed many tests both experimentally and theoretically, the minimal standard model is valid up to scales not so high above the weak scale. In order to describe particle physics at higher energies, the standard model needs to be extended.

The direction of the extension depends on whether the Higgs boson is fundamental or composite. If the Higgs boson is fundamental at higher scales, quantities such as the mass squared would significantly run through a renormalization group flow between the two separate scales. Although this effect may drive the mass squared into a negative value and various conditions may favor a specific negative value, it needs to take into account that the running can technically pass across zero and other negative values. If the Higgs boson is composite, the potential can be suddenly generated. When constituents of the Higgs boson are strongly attracted and form the bound state, some energy would be released. The classical background for the Higgs boson is stabilized at the minimum of the potential. The size of the vacuum expectation value is determined instantly by the condensation without the above problem of the running.

An economical and interesting idea of the composite Higgs boson was given in Ref. [1]. The idea is called the self-breaking of the standard model gauge symmetry. The point is that the gauge bosons of the standard model propagating in extra dimensions can rapidly become strongly coupled and form scalar bound states of quarks and leptons. The authors proposed that the existence of a Higgs doublet is a consequence of the standard model gauge symmetry and three generation of quarks and leptons provided the gauge bosons and fermions propagate in appropriate extra dimensions compactified at a TeV scale. It has also been shown earlier that electroweak symmetry may be broken by fields propagating in extra dimensions [2][3].

In this paper, the idea is applied to a gauge theory in a warped space whose cutoff is much larger than the weak scale or a TeV scale. Gauge couplings of fermions in the warped space and phenomenological constraints have been studied in Ref. [4] and it has been pointed out that brane couplings are large compared to bulk couplings [5]. Bound states in the warped space have been studied in Ref. [6]. Here it has been shown that color-triplet composites have positive masses squared. These results above have been given for fermions with vanishing bulk masses. For bulk fermions, the values of gauge couplings are affected by the masses. The dependence of the bulk gauge couplings with zero-mode fermions on the bulk mass parameters has been given in Ref. [7]. If the fourth generation is introduced in the warped space, it can be the source of a flavor structure as well as the dynamical electroweak symmetry breaking [8]. Therefore a part of the application of the idea to the warped space can be seen from the results in the literature. The other part requires a new analysis.

We systematically examine the bulk and brane gauge couplings of bulk fermions and the brane gauge couplings of brane fermions. Our point is that bulk fermion masses are taken into account and that the ordinary four-dimensional massless gluon is not con-

densed. Unlike Ref. [1] with a strict predictive power for the top quark mass, bulk mass parameters and a brane mixing are included as extra degrees of freedom to realize the pattern of quark and lepton masses. The analysis is performed with the Kaluza-Klein (KK) mode expansion. Binding strengths of fermion constituents are estimated with the most-attractive-channel approximation and the evaluation of the gauge couplings. In particular, color-triplet scalar bound states composed of a quark and a lepton need to be avoided. Once a scenario of the electroweak symmetry breaking is chosen, the weak mixing angle, gauge boson masses and Higgs boson mass are given in terms of the vacuum expectation values.

The paper is organized as follows. In Sec. 2, the action integral and mode functions are given. The boundary values of the mode functions are shown explicitly. In Sec. 3, the field content is given. The binding strengths in the most-attractive-channel approximation are shown. Numerical evaluation for various bulk and brane gauge couplings are given in Sec. 4. A scenario for a condensation to trigger electroweak symmetry breaking is described. In Sec. 5, the gauge boson masses, weak mixing angle and Higgs boson mass are related to the possible vacuum expectation values. We conclude in Sec. 6.

2   Action integral and mode functions

We consider a gauge theory for a bulk fermion Ψ and a brane fermion $\hat{\Psi}$ in a warped space whose metric is given by $ds^2 = e^{-2\sigma}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$ [9] [10] where $\sigma = k|y|$ and $|y| \le L$. Here k denotes the curvature in the five-dimensional anti-de Sitter space. For the fundamental region $0 \le y \le L$, the metric in terms of the coordinate $z = e^{ky}$ is written as $ds^2 = z^{-2}(\eta_{\mu\nu}dx^\mu dx^\nu - k^{-2}dz^2)$. Branes are placed at $z = z_0 = 1$ and $z = z_1 = e^{kL} \equiv z_L$. The starting action integral is given by[*]

$$
I = \int d^4x \int_0^L dy \sqrt{\det(g_{KL})}\,\mathrm{tr}\left[-\frac{1}{2}F_{MN}F_{PQ}g^{MP}g^{NQ} - \frac{1}{\xi}\omega(A)^2\right]
$$
$$
+ \int d^4x \int_0^L dy \sqrt{\det(g_{KL})}\,\bar{\Psi}iD\Psi + u_i \int d^4x \int_0^L dy \sqrt{-\det(g_{\rho\sigma})}\,\bar{\Psi}iD\Psi\delta(y-y_i)
$$
$$
+ \int d^4x \int_0^L dy \sqrt{-\det(g_{\rho\sigma})}\,\bar{\hat{\Psi}}iDP_c\hat{\Psi}\delta(y-y_i), \tag{2.1}
$$

where u_i are dimensionful coefficients and P_c is a chirality projection. The covariant derivative acts as

$$
D\Psi = \left\{\Gamma^A e_A^M(\partial_M + \frac{1}{8}\omega_{MBC}\left[\Gamma^B,\Gamma^C\right] - ig_A A_M) + ic\sigma'\right\}\Psi, \tag{2.2}
$$

$$
DP_c\hat{\Psi} = \Gamma^a e_a^\mu(\partial_\mu - ig_A A_\mu)P_c\hat{\Psi}, \qquad \Gamma^a = \begin{pmatrix} 0 & \sigma^a \\ \bar{\sigma}^a & 0 \end{pmatrix}, \quad \Gamma^5 = i\gamma^5 \tag{2.3}
$$

where the spin connection and the bulk mass parameter are denoted as $\omega_{\mu a 5} = -\sigma' e^{-\sigma}\delta_{\mu a}$ and c, respectively. Here $\{\Gamma^a, \Gamma^b\} = 2\eta^{ab}$, $\eta = \mathrm{diag}(1,-1,-1,-1)$ and $\gamma^5 = \mathrm{diag}(-1,1)$.

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[*]Even in the flat bulk space, the five-dimensional Lorentz invariance is violated at a quantum level for orbifolds [11]. A further generalization in this direction is left for future work.

The gauge fixing functional is given by $\omega(A) = \partial_\mu A_\nu \cdot g^{\mu\nu} + \xi z g^{zz} \partial_z (A_z/z)$. The extension for multiple fields is straightforward. For the gauge group SU(3) $\times$ SU(2) $\times$ U(1), the covariant derivative is given by

$$D\Psi = \left\{ \Gamma^A e_A^M (\partial_M + \frac{1}{8}\omega_{MBC}\left[\Gamma^B, \Gamma^C\right] - ig_3 G_M - ig_2 W_M - ig_1 Y B_M) + ic\sigma' \right\} \Psi. \quad (2.4)$$

The gauge fields will be denoted as $A_M = \{G_M, W_M, B_M\}$, collectively.

In the action integral (2.1), terms for four fermion interactions are not included. Such terms may be induced by strongly-coupled effects. Once Ψ^2 or $\hat{\Psi}^2$ is regarded as a composite scalar Φ or $\hat{\Phi}$, the coefficient of Φ^2 or $\hat{\Phi}^2$ term is fixed by the condensation without knowing the value of the coefficient of the original Ψ^4 or $\hat{\Psi}^4$ term. Another aspect of the action integral is that the bulk fields have the bulk and brane terms. In addition to bulk fermions, brane fermions contributions to brane terms. Both of the gauge couplings for the bulk fermion Ψ and the brane fermion $\hat{\Psi}$ are flavor diagonal. A part of fields Ψ and $\hat{\Psi}$ can become heavy and the other can become light, depending on the quantum numbers. This gives rise to a flavor mixing [13][14]. Furthermore, the coefficient of the brane kinetic term is not an ambiguous parameter. This is contrastive to the coefficients of irrelevant operators which are ambiguous and unfixed within the present framework. For example, one of ways to achieve the coefficient is to employ the property of chirality. The on-shell renormalization requires a propagator to evenly decompose for two chiralities while corrections on orbifolds differ between left- and right-handed fermions. Hence, the two independent renormalization coefficients are required. In the flat space, the wave function renormalization and the coefficient of the brane kinetic term are shown to be unambiguously fixed [12]. For simplicity, the brane kinetic term and the Ψ-$\hat{\Psi}$ mixing will be omitted in the present analysis whereas brane gauge couplings will be taken into account for both of Ψ and $\hat{\Psi}$.

We examine the case where the only possible source of gauge symmetry breaking is condensation. The boundary conditions are imposed as Neumann for the 4-component A_μ and Dirichlet for the extra-dimensional component A_y. The boundary conditions for fermions will be assigned depending on their quantum numbers. Identical boundary conditions are adopted at the positions of the two branes. For example, a fermion with the Neumann condition for the left-handed component at $y = 0$ obeys the Neumann condition for $y = L$. Then the left-handed component has a zero mode. In the $\xi = 1$ gauge, the gauge field is decomposed into

$$A_\mu(x,z) = \frac{1}{\sqrt{L}} \left[A_{\mu 0}(x) + \sum_{n=1}^\infty N_n A_{\mu n}(x)\chi_n(z) \right], \quad (2.5)$$

$$A_z(x,z) = \frac{1}{k\sqrt{L}} \sum_{n=1}^\infty N_n A_{zn}(x)\phi_n(z). \quad (2.6)$$

The mode functions are given by

$$\chi_n = z F_{1,0}(m_n z/k, m_n z_L/k), \qquad \phi_n = z F_{0,0}(m_n z/k, m_n z_L/k). \quad (2.7)$$

Here the function F is defined as $F_{\alpha,\beta}(u,v) \equiv J_\alpha(u)Y_\beta(v) - Y_\alpha(u)J_\beta(v)$ for non-negative u, v. It satisfies $F_{-n,-m}(u,v) = (-1)^{n+m}F_{n,m}(u,v)$ for $n, m = 0, 1, 2, \cdots$, and

$$F_{-\gamma,-\delta}(u,v) = \sin[(\delta - \gamma)\pi] \cdot [J_\gamma(u)J_\delta(v) + Y_\gamma(u)Y_\delta(v)] + \cos[(\delta - \gamma)\pi] \cdot F_{\gamma,\delta}(u,v),$$

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where γ, δ are not integers. Hence, $F_{-\alpha,-\alpha}(u,v) = F_{\alpha,\alpha}(u,v)$. The n-th KK gauge boson mass is obtained from the eigenvalue equation

$$F_{0,0}(m_n/k, m_n z_L/k) = 0. \tag{2.8}$$

The normalization is given by $N_n^{-2} = \int_1^{z_L} dz\, \chi_n^2/(kLz)$.

Except for the δ function part, the equations of motion for fermions are given by

$$i\sigma \cdot \partial \tilde{\Psi}_R + k D_+ \tilde{\Psi}_L = 0, \qquad i\bar{\sigma} \cdot \partial \tilde{\Psi}_L - k D_- \tilde{\Psi}_R = 0, \tag{2.9}$$

where $\tilde{\Psi} \equiv z^{-2} k^{-1/2} \Psi$. The bulk fermion is decomposed into

$$\tilde{\Psi}(x,z) = f_0(z)\psi_0(x) + \sum_{n=1}^{\infty} N_{fn} f_n(z)\psi_n(x). \tag{2.10}$$

The normalization is given by $N_{fn}^{-2} = \int_1^{z_L} dz\, f_n^2$. The mode functions obey

$$(D_+ D_- - m_n^2/k^2) f_{Rn} = 0, \qquad (D_- D_+ - m_n^2/k^2) f_{Ln} = 0. \tag{2.11}$$

Here $D_+ \equiv z^c \partial_z z^{-c}\cdot$ and $D_- \equiv -z^{-c} \partial_z z^c\cdot$. The δ function part depends on the boundary conditions at $z = z_i$.

R-even

When the right-handed component obeys the Neumann boundary condition, we call its fermion an R-even fermion and the δ function part means $D_- \tilde{\Psi}_R \big| = 0$ at the boundary. The mode functions are given by

$$\text{even} \quad f_{Rn} = \sqrt{z} F_{c+(1/2),c-(1/2)}(m_n z/k, m_n z_L/k), \tag{2.12}$$
$$\text{odd} \quad f_{Ln} = \sqrt{z} F_{c-(1/2),c-(1/2)}(m_n z/k, m_n z_L/k). \tag{2.13}$$

The left-handed component has the Dirichlet boundary condition. The zero mode exists for an even component, $f_{R0} = [(1-2c)/(z_L^{1-2c} - 1)]^{1/2} z^{-c}$. The eigenvalue equation for the n-th KK fermion is given by

$$F_{c-(1/2),c-(1/2)}(m_n/k, m_n z_L/k) = 0. \tag{2.14}$$

The mass for $(c-1/2)$ is the same as the mass for $-(c-1/2)$. For example, the mass for $c = 2$ is the same as the mass for $c = -1$. The eigenvalue equation for a KK gauge boson, (2.8) is the same as for an R-even fermion with $c = 1/2$.

L-even

When the left-handed component obeys the Neumann boundary condition, we call its fermion an L-even fermion and the δ function part means $D_+ \tilde{\Psi}_L \big| = 0$ at the boundary. The mode functions are given by

$$\text{even} \quad f_{Ln} = \sqrt{z} F_{c-(1/2),c+(1/2)}(m_n z/k, \lambda m_n z_L/k), \tag{2.15}$$
$$\text{odd} \quad f_{Rn} = \sqrt{z} F_{c+(1/2),c+(1/2)}(m_n z/k, m_n z_L/k). \tag{2.16}$$

The right-handed fermion has the Dirichlet boundary condition. The zero mode exists for an even component, $f_{L0} = [(1+2c)/(z_L^{1+2c} - 1)]^{1/2} z^c$. The eigenvalue equation for the n-th KK fermion is given by

$$F_{c+(1/2),c+(1/2)}(m_n/k, m_n z_L/k) = 0. \tag{2.17}$$

From the property of F, this equation equals $F_{-c-(1/2),-c-(1/2)}(m_n/k, m_n z_L/k) = 0$, which is Eq. (2.14) with $c \leftrightarrow -c$. Therefore, the mass for an L-even fermion with c is equal to the mass for an R-even fermion with $-c$.

KK masses are calculated numerically. One input is the warp factor. The assumption is that the five-dimensional geometry is a classical background. We regard the cutoff at $y = 0$ as an intermediate scale which is much lower than the Planck scale or the stabilization of geometry as in Ref. [15]. The warp factor is adopted as $z_L = 10^{10}$. In order that the KK scale $m_{KK} = \pi k/(z_L - 1)$ is of the order $\mathcal{O}(1)$TeV, as the other input the curvature is taken as $k = 4 \times 10^{12}$. Then KK masses are given with the parameter c in Table 1.

Table 1: KK masses in unit of GeV. The fermion masses are shown for R-even fermions.

KK mode	1	2	3	4	5	c
Gauge boson	9896.51	22372	34913.4	47469.3	60030.4	$(1/2)$
Fermion	23053.8	36380	49291.8	62058.4	74756.1	-2
	17973.6	30901	43616.5	56264.8	68883	-1
	12566.4	25132.7	37699.1	50265.5	62831.9	0

For the same input values, the boundary values for the mode functions are shown in Table 2 and Fig. 1. The mode functions for the gauge boson have small values for $y = 0$

Table 2: Boundary value $N_n \chi_n|_{y=0,L}$.

KK mode	1	2	3	4	5
$y = 0$	-0.24066	0.167176	-0.136784	0.119079	-0.107114
$y = L$	6.79041	6.7882	6.78752	6.78719	6.78699

and large values for $y = L$. If there are gauge couplings on the brane with a TeV-scale cutoff, their values tend to receive large contributions from the gauge boson part.

The boundary values of mode functions for fermions depend on whether they are KK modes or zero mode. When they are KK modes, the dependence on the KK level is not large. For $y = L$, the boundary values are small and almost independent of c. For $y = 0$, they are also small in a wide region. On the other hand, zero mode significantly depends on c. For a positive and large c, the boundary value is small. The mode function for a R-even fermion, f_R is given by f_L with the replacement $c \to -c$. For a positive and large c, the boundary value for R-even f_{R0} is large.

In the next section, we will consider composite objects with the form $\bar{\Psi}_1 \Psi_2$. If the constituent Ψ_1 with a mass parameter c_1 is an L-even fermion and Ψ_2 with c_2 is an R-even fermion, $\bar{\Psi}_1 \Psi_2$ has zero mode. After the condensation, the composite scalar can
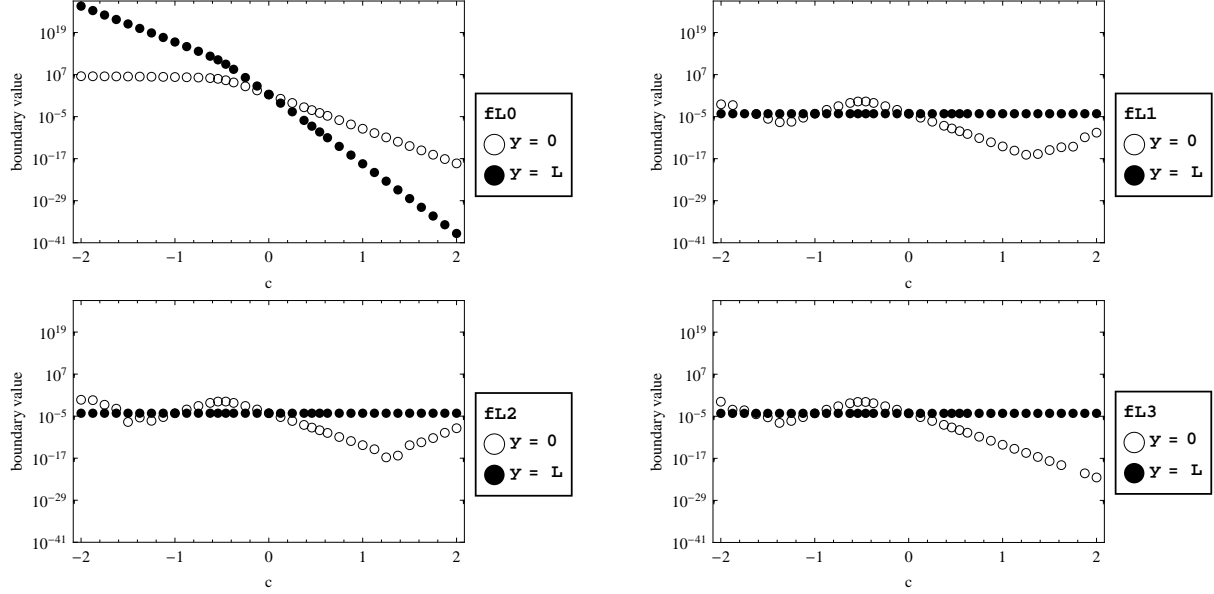
Figure 1: Boundary value $f_{Ln}|_{y=0,L}$ where $n = 0, 1, 2, 3$.

give rise to a potential with a negative mass squared as pointed out in Refs. [1][6]. The y-dependent profile of the zero mode for $\bar{\Psi}_1 \Psi_2$ is given by $\bar{\Psi}_{1L0} \Psi_{2R0} \sim z_L^{-(c_1-c_2)y/L}$. The localized position for the zero mode contributions of the composite is determined by the difference $c_1 - c_2$. For $c_1 > c_2$, they are localized at the brane with an intermediate-scale cutoff. It is an open question whether the condensation localized at the brane with an TeV-scale cutoff significantly affects low energy physics. However, a scenario to respect the standard model at low energies is a high-energy condensation and it may be natural to expect that it corresponds to $c_1 \geq c_2$. If all we have to do is to reproduce the pattern of observed masses, it might be fulfilled even for $c_1 = c_2$ for five-dimensional Dirac fermions corresponding to weak-doublet and singlet fields such as e_L and e_R [16].

3 Matter content and attractive force

Our standpoint is to systematically examine attractive forces for various possible fermions rather than to build a model with a fixed set of fermions. The field contents are similar to the standard model based on SU(3) × SU(2) × U(1). One generation is treated. Instead of the four-dimensional chiral fermions, fields are bulk Dirac fermions and brane chiral fermions. The fields and their quantum numbers are given in Table 3. For the bulk fields, each zero mode is shown.

The SU(2)-doublet bulk fermions Q, L are L-even fermions and the singlet bulk fermions U, D, N, E are R-even fermions. Each Dirac fermion has the masses c_f where $f = Q, L, U, D, N, E$. These parameters generate the hierarchical pattern of the observed masses. Following Ref. [1], we perform the most-attractive-channel approximation. The coefficient of the potential for the attractive force for $\bar{\Psi}_1 \Psi_2$ is given by [17]

$$\frac{1}{2} \left[C_2(\bar{\Psi}_1) + C_2(\Psi_2) - C_2(\bar{\Psi}_1 \Psi_2) \right], \tag{3.1}$$

6

Table 3: Bulk and brane matter

5D bulk field (zero mode)	Quantum number	4D brane field
$Q_0 = \begin{pmatrix} t \\ b \end{pmatrix}_L$	$(3, 2, \frac{1}{6})$	$\hat{Q} = \begin{pmatrix} \hat{t} \\ \hat{b} \end{pmatrix}_L$
$U_0 = t_R$	$(3, 1, \frac{2}{3})$	$\hat{U} = \hat{t}_R$
$D_0 = b_R$	$(3, 1, -\frac{1}{3})$	$\hat{D} = \hat{b}_R$
$L_0 = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$(1, 2, -\frac{1}{2})$	$\hat{L} = \begin{pmatrix} \hat{\nu}_\tau \\ \hat{\tau} \end{pmatrix}_L$
$N_0 = \nu_{\tau R}$	$(1, 1, 0)$	$\hat{N} = \hat{\nu}_{\tau R}$
$E_0 = \tau_R$	$(1, 1, -1)$	$\hat{E} = \hat{\tau}_R$

where $C_2(r)$ is the second Casimir invariant for the representation r of the gauge group. Possible combinations for $\bar{\Psi}_1 \Psi_2$ for bulk fermions are shown in Table 4. Each combination

Table 4: Binding strength for $\bar{\Psi}_1 \Psi_2$.

	Constituents	SU(3)$\times$ SU(2)$\times$U(1) representation	Binding strength	Relative binding for $\sqrt{\frac{3}{5}}\, g_1 = g_2 = g_3$
H_1	$\bar{Q}U$	$(1, 2, \frac{1}{2})$	$\frac{4}{3}g_3^2 + \frac{1}{9}g_1^2$	1
H_2	$\bar{Q}D$	$(1, 2, -\frac{1}{2})$	$\frac{4}{3}g_3^2 - \frac{1}{18}g_1^2$	0.93
S_1	$\bar{Q}Q$	$(1, 1, 0)$	$\frac{4}{3}g_3^2 + \frac{3}{4}g_2^2 + \frac{1}{32}g_1^2$	1.5
S_2	$\bar{U}U$	$(1, 1, 0)$	$\frac{4}{3}g_3^2 + \frac{4}{9}g_1^2$	1.14
S_3	$\bar{D}D$	$(1, 1, 0)$	$\frac{4}{3}g_3^2 + \frac{1}{9}g_1^2$	1
S_4	$\bar{U}D$	$(1, 1, -1)$	$\frac{4}{3}g_3^2 - \frac{2}{9}g_1^2$	0.86
S_5	$\bar{L}L$	$(1, 1, 0)$	$\frac{3}{4}g_2^2 + \frac{1}{4}g_1^2$	0.64
S_6	$\bar{E}E$	$(1, 1, 0)$	g_1^2	0.21
S_7	$\bar{L}Q$	$(3, 1, \frac{2}{3})$	$\frac{3}{4}g_2^2 - \frac{1}{12}g_1^2$	0.5
S_8	$\bar{E}U$	$(3, 1, \frac{5}{3})$	$-\frac{2}{3}g_1^2$	-0.29
S_9	$\bar{E}D$	$(3, 1, \frac{2}{3})$	$\frac{1}{3}g_1^2$	0.14
S_{10}	$\bar{E}Q$	$(3, 2, \frac{7}{6})$	$-\frac{1}{6}g_1^2$	-0.07
S_{11}	$\bar{L}U$	$(3, 2, \frac{5}{6})$	0	0
S_{12}	$\bar{L}D$	$(3, 2, \frac{1}{6})$	$\frac{1}{6}g_1^2$	0.07
S_{13}	$\bar{L}E$	$(1, 2, -\frac{1}{2})$	$\frac{1}{2}g_1^2$	0.21
S_{14}	N or $\bar{N}$ is included		0	0

can contain other degrees of freedom with respect to quantum numbers. A large quantum number gives rise to a large negative contribution which is the last term in Eq. (3.1). These degrees of freedom are omitted in Table 4. For example, $\bar{Q}Q$ includes (1,3,0). The binding strength for (1,3,0) is weaker than that of (1,1,0).

The field H_1 has the quantum number for a Higgs doublet and H_2 corresponds to its dagger. The fields $S_1, \cdots, S_9$ have a mixed form for chirality such as the decomposition $\bar{Q}_L Q_R$ and $\bar{Q}_R Q_L$. One of the decomposition, Q_R for Q does not have zero mode. Even

if $\bar{Q}Q$ becomes a scalar bound state, its potential would not give rise to a negative mass squared. The other combinations $S_{10}, \cdots S_{14}$ have zero modes but binding strengths are relatively small. If the net couplings of the constituents for S_{12} and S_{13} remain small, the candidates of composite scalars to potentially yield a vacuum expectation value would be only H_1 and H_2.

For brane fermions, there are no correspondents for $S_1, \cdots, S_9$. Binding strengths for brane fermions can be found in an analogous way to the list given in Table 4.

We assume that the ordinary gluon which is massless in four dimensions does not lead to any condensation. Therefore, our point for a condensation is whether couplings are clearly large compared to gluon couplings. Numerical analysis for the four-dimensional effective couplings will be performed in the next section.

4 Numerical analysis for couplings

The gauge couplings are included in covariant derivatives in the action (2.1). The four-dimensional effective gauge interactions for zero-mode and KK-mode fields are given by

$$
\int d^4x \sum_{n,m,\ell} \left(g^{\text{bulk}}_{L,nm\ell} \bar{\tilde{\Psi}}_{Ln} \gamma \cdot A_m \tilde{\Psi}_{L\ell} + g^{\text{bulk}}_{R,nm\ell} \bar{\tilde{\Psi}}_{Rn} \gamma \cdot A_m \tilde{\Psi}_{R\ell} \right)
$$

$$
+ \int d^4x \sum_{n,m,\ell} \left(g^{\text{brane}}_{L,nm\ell} \bar{\tilde{\Psi}}_{Ln} \gamma \cdot A_m \tilde{\Psi}_{L\ell} + g^{\text{brane}}_{R,nm\ell} \bar{\tilde{\Psi}}_{Rn} \gamma \cdot A_m \tilde{\Psi}_{R\ell} \right)
$$

$$
+ \int d^4x \sum_m g_m \left(\bar{\tilde{\Psi}}_L \gamma \cdot A_m P_c \hat{\Psi}_L + \bar{\tilde{\Psi}}_R \gamma \cdot A_m P_c \hat{\Psi}_R \right). \tag{4.1}
$$

Here the four-dimensional effective gauge couplings are given in terms of z-integral by

$$
g^{\text{bulk}}_{L,nm\ell} = \frac{g_A}{\sqrt{L}} N_{Ln} N_m N_{L\ell} \int_1^{z_L} dz\, f_{Ln} \chi_m f_{L\ell}, \tag{4.2}
$$

$$
g^{\text{bulk}}_{R,nm\ell} = \frac{g_A}{\sqrt{L}} N_{Rn} N_m N_{R\ell} \int_1^{z_L} dz\, f_{Rn} \chi_m f_{R\ell}, \tag{4.3}
$$

$$
g^{\text{brane}}_{L,nm\ell} = u_i k \frac{g_A}{\sqrt{L}} N_{Ln} N_m N_{L\ell} \int_1^{z_L} dz\, f_{Ln} \chi_m f_{L\ell}\, z\delta(z - z_i), \tag{4.4}
$$

$$
g^{\text{brane}}_{R,nm\ell} = u_i k \frac{g_A}{\sqrt{L}} N_{Rn} N_m N_{R\ell} \int_1^{z_L} dz\, f_{Rn} \chi_m f_{R\ell}\, z\delta(z - z_i), \tag{4.5}
$$

$$
g_m = \frac{g_A}{\sqrt{L}} N_m \int_1^{z_L} \frac{dz}{z^3} \chi_m \delta(z - z_i). \tag{4.6}
$$

We focus on zero mode and the first three KK modes. For each KK level, there are the corresponding couplings $g_{n0\ell}$, g_{0m0}, g_{nmn}, g_{0mn}, $g_{nm\ell}$ where $n, m, \ell = 1, 2, 3$ and the repetition of the same letter such as g_{nmn} does not mean the summation. Bulk and brane couplings are given for L- and R-even fermions in the following.

Bulk couplings

For a zero-mode gauge boson, the bulk couplings are given by $g^{\text{bulk}}_{L,n0\ell} = g^{\text{bulk}}_{R,n0\ell} = \delta_{n\ell} g_A/\sqrt{L}$ where $n, \ell = 0, 1, 2, \cdots$. The coupling of zero-mode fermions with a gluon is $g_A/\sqrt{L}$. The

condensation of our interest should have strong couplings compared to this value. The summation of KK mode also needs to be taken into account.

For zero-mode fermions, its coupling with the first few KK-mode gauge bosons is given in Fig. 2.[†] The figure is shown for L-even fermions.
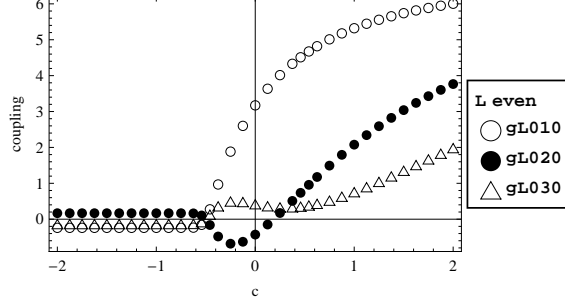

Figure 2: Couplings $g_{L,0m0}^{\mathrm{bulk}}$ divided by $g_A/\sqrt{L}$, where the fermions are L even and the KK modes of the gauge bosons are $m = 1, 2, 3$.

For $c < -1/2$, the couplings are small and their c-dependence is small. For $c > -1/2$, the size of the couplings is rapidly enhanced. The couplings with the first KK-mode tend to be large. In a wide region for c, $|g_{L,010}^{\mathrm{bulk}}| > |g_{L,020}^{\mathrm{bulk}}| > |g_{L,030}^{\mathrm{bulk}}|$. The existence of an evident inequality suggests the convergence of the summation for KK modes. In any case, for $c > -1/2$ the couplings become strong compared to the gluon coupling. For R-even fermions, the couplings $g_{R,0m0}^{\mathrm{bulk}}$ are given by $g_{L,0m0}^{\mathrm{bulk}}$ with the replacement $c \leftrightarrow -c$.

The bulk couplings with KK fermions, $g_{L,nmn}^{\mathrm{bulk}}$ are shown in Fig. 3. Here the two fermions have identical KK levels. For both of R-even and L-even fermions, the couplings with the first KK gauge boson are large independently of c, $g_{L,n1n}^{\mathrm{bulk}} \gtrsim 3$. Unlike $g_{L,0n0}^{\mathrm{bulk}}$ which significantly depends on c, it seems inevitable that the couplings of KK fermions with the first KK gauge boson are strong compared to the gluon coupling. On the other hand, it is read that $|g_{Ln1n}^{\mathrm{bulk}}| > |g_{Ln2n}^{\mathrm{bulk}}|, |g_{Ln3n}^{\mathrm{bulk}}|$. This is an evident inequality similarly to the case of g_{L0n0}^{bulk}.

From the couplings for left-handed fermions, $g_{L,nmn}^{\mathrm{bulk}}$, the couplings for right-handed fermions, $g_{R,nmn}^{\mathrm{bulk}}$ can be obtained. The couplings $g_{R,1m1}^{\mathrm{bulk}}, g_{R,2m2}^{\mathrm{bulk}}, g_{R,3m3}^{\mathrm{bulk}}$ for L-even fermions have the same values as $g_{L,1m1}^{\mathrm{bulk}}, g_{L,2m2}^{\mathrm{bulk}}, g_{L,3m3}^{\mathrm{bulk}}$ for R-even fermions with the replacement $c \leftrightarrow -c$, respectively where $m = 1, 2, 3$. Similarly, R-even $g_{R,1m1}^{\mathrm{bulk}}, g_{R,2m2}^{\mathrm{bulk}}, g_{R,3m3}^{\mathrm{bulk}}$ have the same values as L-even $g_{L,1m1}^{\mathrm{bulk}}, g_{L,2m2}^{\mathrm{bulk}}, g_{L,3m3}^{\mathrm{bulk}}$ with the replacement $c \leftrightarrow -c$, respectively.

It is also possible that only one of fermions in couplings is zero mode. Zero modes belong to the right-handed component for an R-even fermion and the left-handed component for an L-even fermion. R-even $g_{L,0mn}^{\mathrm{bulk}}$ and L-even $g_{R,0mn}^{\mathrm{bulk}}$ are vanishing or do not exist where $n \neq 0$. Non-vanishing couplings are shown in Fig. 4. The couplings have a region with a significant c-dependence and a region almost independent of c. In this behavior, the couplings with one zero-mode fermion resemble $|g_{L,0m0}^{\mathrm{bulk}}|$ and $|g_{R,0m0}^{\mathrm{bulk}}|$ as we have seen in Fig. 2.

[†]The overall sign for $g_{L,020}^{\mathrm{bulk}}$ seems opposite to that of Ref. [7]. This is due to the difference of the normalization.
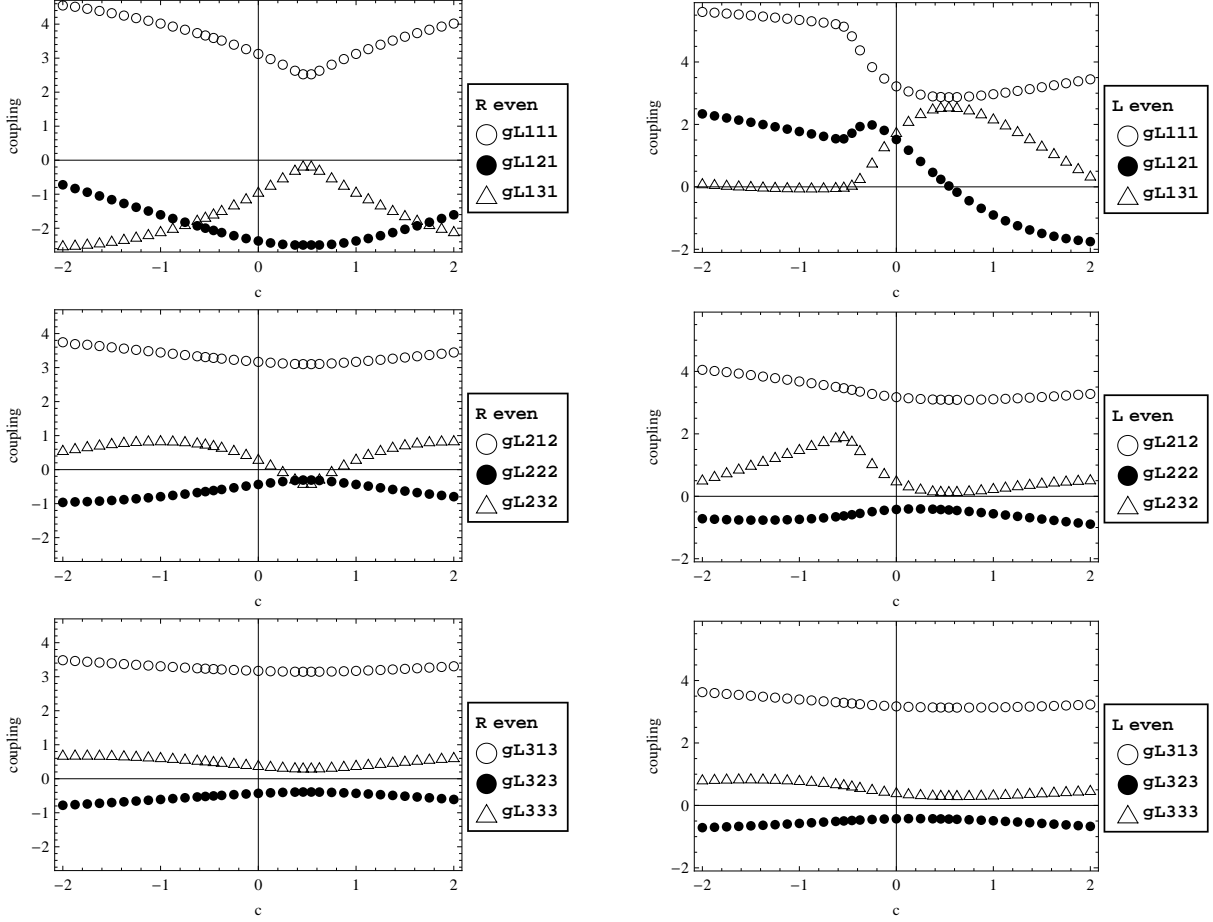
Figure 3: Couplings $g_{L,nmn}^{\text{bulk}}$ divided by $g_A/\sqrt{L}$, where the fermions have identical KK modes $n = 1, 2, 3$.

As $g_{L,010}^{\text{bulk}} \neq 0$, the KK number conservation is violated. Furthermore, $g_{L,010}^{\text{bulk}} > g_{L,000}^{\text{bulk}} = g_A/\sqrt{L}$ means that conserved quantities are not even supported more than violated quantities. However, among other couplings, $|g_{R,022}^{\text{bulk}}| > |g_{R,012}^{\text{bulk}}|$ and $|g_{R,033}^{\text{bulk}}| > |g_{R,013}^{\text{bulk}}|, |g_{R,023}^{\text{bulk}}|$ seem to support that the KK number conservation is favored. If this shows that there exists a critical m for the largest coupling for each of $|g_{R,0m1}^{\text{bulk}}|, |g_{R,0m2}^{\text{bulk}}|, |g_{R,0m3}^{\text{bulk}}|$, it may be related to the convergence for the summation for KK modes. For $g_{R,0m0}^{\text{bulk}}$ and $g_{R,0mn}^{\text{bulk}}$, it is interesting that the couplings $|g_{R,010}^{\text{bulk}}|, |g_{R,021}^{\text{bulk}}|$ and $|g_{R,032}^{\text{bulk}}|$, in other words, $|g_{R,0(n+1)n}^{\text{bulk}}|$, are large in a significantly c-dependent region.

For L-even fermions, the couplings $g_{L,0mn}^{\text{bulk}}$ are given by the values with the same size and opposite sign as the R-even $g_{R,0mn}^{\text{bulk}}$ with the replacement $c \leftrightarrow -c$ where $m, n = 1, 2, 3$.

The other couplings are KK gauge couplings for fermions with different KK modes. The $g_{L,nm\ell}^{\text{bulk}}$ and $g_{R,nm\ell}^{\text{bulk}}$ with $n, m, \ell = 1, 2, 3$ and $n \neq \ell$ are shown for R-even fermions in Fig. 5. The profile of R-even $g_{L,nm\ell}^{\text{bulk}}$ resembles that of R-even $g_{L,nmn}^{\text{bulk}}$. For R-even fermions, in wide regions $|g_{L,122}^{\text{bulk}}| > |g_{L,132}^{\text{bulk}}| > |g_{L,112}^{\text{bulk}}|, |g_{L,133}^{\text{bulk}}| > |g_{L,123}^{\text{bulk}}| > |g_{L,113}^{\text{bulk}}|, |g_{L,223}^{\text{bulk}}| > |g_{L,213}^{\text{bulk}}| > |g_{L,233}^{\text{bulk}}|$. All the largest couplings above fulfill
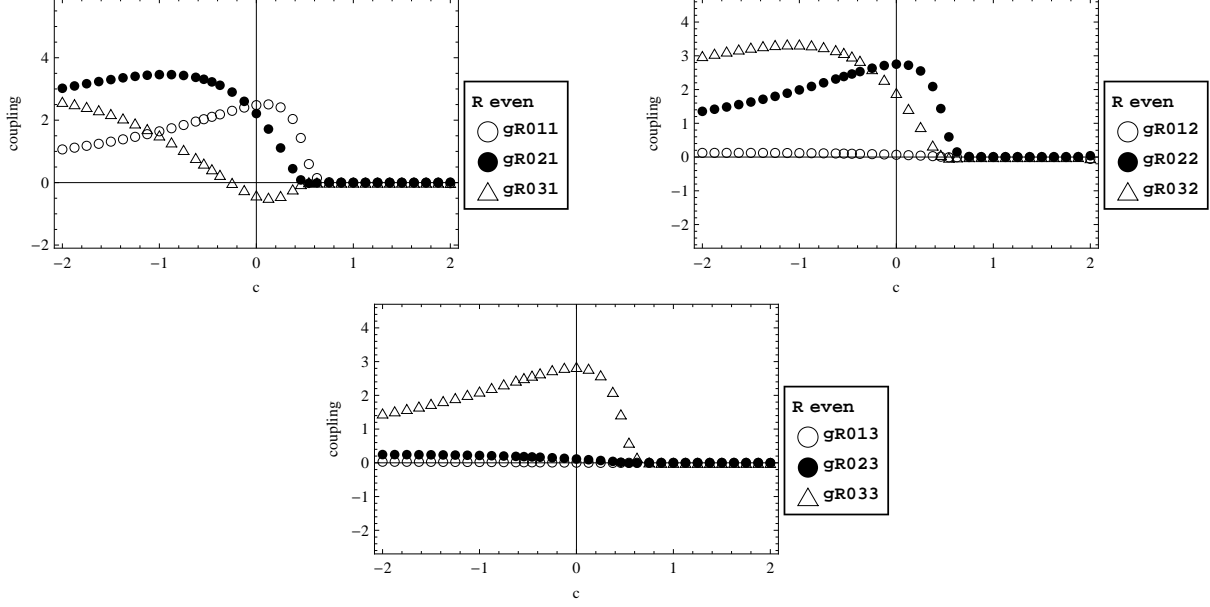
$$\ell - n + 1 = m, \tag{4.7}$$

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Figure 4: Couplings $g_{R,0mn}^{\rm bulk}$ divided by $g_A/\sqrt{L}$, where the fermions are R even. One fermion is zero mode and the others are KK modes $n = 1, 2, 3$.

for the KK levels. This relation is fulfilled also for $g_{R,0(n+1)n}^{\rm bulk}$ which are large couplings as found above. It is remarkable that the KK level relation given in Eq. (4.7) is not the KK mode conservation.

The values of the couplings can be larger than that of the gluon coupling independently of c. This behavior is the same for $g_{R,nm\ell}^{\rm bulk}$. For L-even fermions, $g_{L,nm\ell}^{\rm bulk}$ and $g_{R,nm\ell}^{\rm bulk}$ is given by $g_{R,nm\ell}^{\rm bulk}$ and $g_{L,nm\ell}^{\rm bulk}$ with the replacement $c \leftrightarrow -c$, respectively.

In summary for bulk couplings, the ordinary gluon, or the four-dimensional massless gluon couples to fermions with at most $g_A/\sqrt{L}$ where fermions are not only zero mode but also KK modes. Therefore, the zero-mode gluon is not the dominant part for a condensation even if the components of fermions are KK modes. A KK-mode gluon can have large couplings compared to $g_A/\sqrt{L}$. When at least one of fermions is zero mode, the gauge couplings can be small. For L-even fermions with $c < -1/2$, the KK gauge couplings have $g_{L,0mn}^{\rm bulk} < g_A/\sqrt{L}$ where $m = 1, 2, 3$ and $n = 0, 1, 2, 3$. For R-even fermions with $c > 1/2$, the KK gauge couplings have $g_{R,0mn}^{\rm bulk} < g_A/\sqrt{L}$. Hence, for L-even fermions with $c < -1/2$ and R-even fermions with $c > 1/2$, the zero modes of a gluon and fermions do not give rise to a condensation solely even if the KK modes of the gluon and fermions are included in the couplings. Without zero modes, KK gauge couplings such as $g_{L,111}^{\rm bulk}$ have large values compared to $g_A/\sqrt{L}$ independently of c. In other words, a condensation may be an inevitable effect composed of KK-mode fields.

This evaluation is applied to the binding strengths given in Table 4. The maximum absolute value of the couplings is about $6g_A/\sqrt{L}$ for $-2 < c < 2$. Even after the contributions are summed with respect to KK modes, S_{12} may be small on account of the smallness of the relative strength. We will see that the constraint for S_{12} mainly arises from the brane coupling rather than the bulk coupling. On the other hand, S_{13} can have
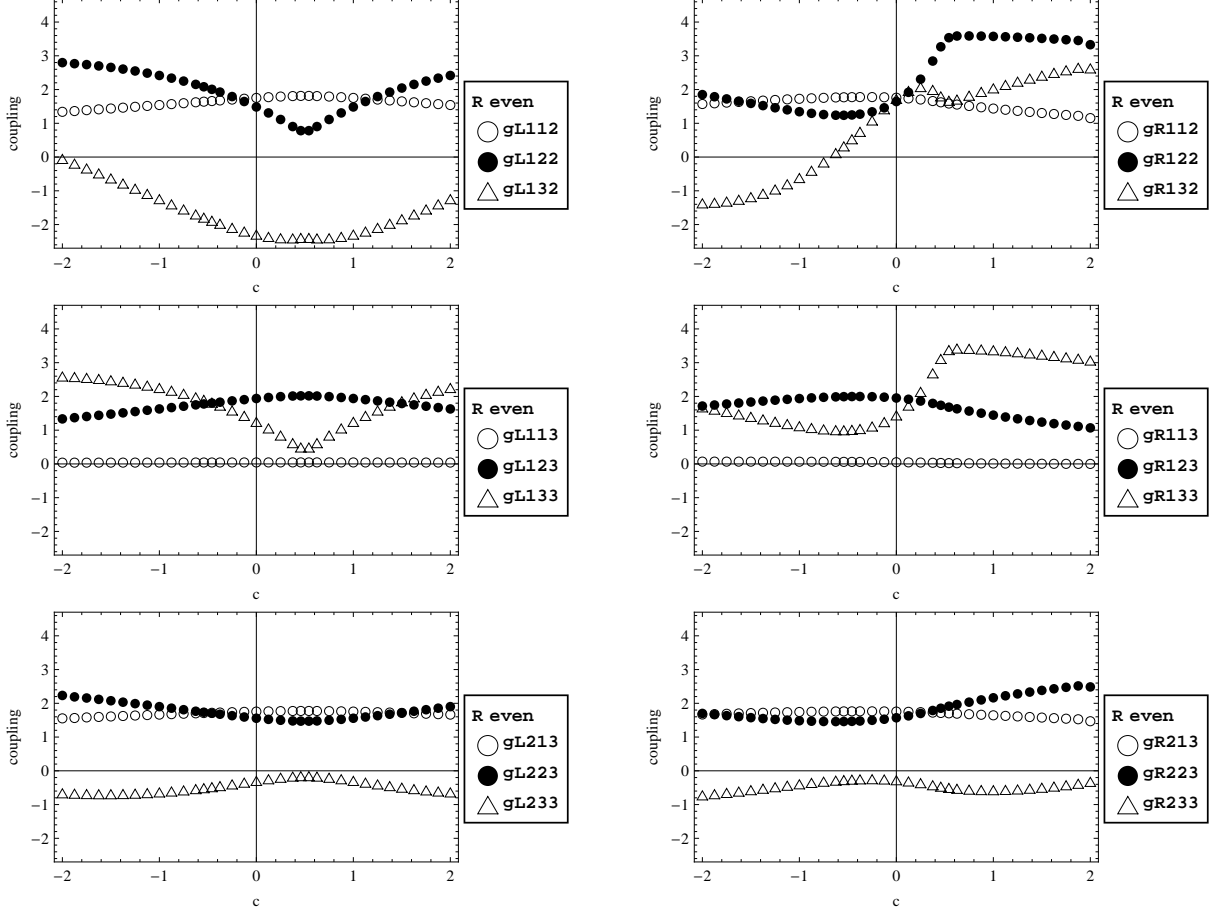
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Figure 5: Couplings $g_{L,nm\ell}^{\text{bulk}}$ and $g_{R,nm\ell}^{\text{bulk}}$ divided by $g_A/\sqrt{L}$, where the fermions have different KK modes $n = 1, 2, 3$ and $\ell = 1, 2, 3$.

large values. The quantum number of S_{13} is the same as that of H_2. It may yield a mixing. For bulk fields, SU(3)-singlet and SU(2)-doublet H_1, H_2 and S_{13} are candidates of composite scalars.

Brane couplings

The boundary values of the mode functions have been found in the end of Sec. 2. The brane couplings (4.4)-(4.6) are given by the products of the boundary values of the mode functions. We write down the couplings for L-even fermions in the following. For R-even fermions, the couplings are given by the corresponding couplings for L-even fermions with the replacement $c \leftrightarrow -c$.

For L-even fermions, the c-dependence of $g_{L,nmn}^{\text{brane}}$ divided by $(u_i/L)g_A/\sqrt{L}$ is shown in Fig. 6. The values of $g_{L,n0n}^{\text{brane}}$ and $g_{L,n1n}^{\text{brane}}$ are not sensitive to the level of KK modes for fermions. As seen in Table 2, this is because the boundary values $N_n\chi_n\big|$ depend on the KK level weakly. From the values in Table 2, the values of $g_{L,n2n}^{\text{brane}}$ and $g_{L,n3n}^{\text{brane}}$ can also be obtained.

For $c \gtrsim 1/2$, the couplings $g_{L,0m0}^{\text{brane}}$ are small, $g_{L,0m0}^{\text{brane}} < (u_i/L)g_A/\sqrt{L}$ at both of $y = 0$
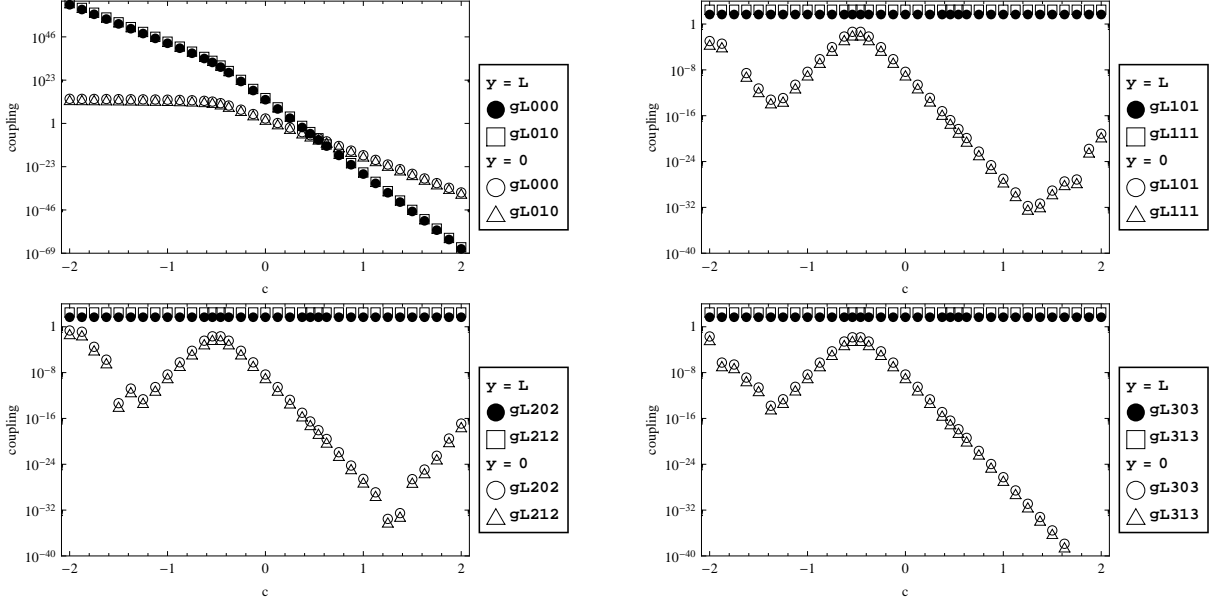
Figure 6: Coupling $g_{L,nmn}^{\text{brane}}$ divided by $(u_i/L)g_A/\sqrt{L}$, where the fermions are L even and have identical KK modes $n = 0, 1, 2, 3$.

and $y = L$ where $m = 0, 1, 2, 3$. In particular, the brane coupling of zero modes of L-even fermions with zero mode of a gluon is given by $g_{L,000}^{\text{brane}}$. The value at $y = L$ can be very large for fermions with $c \lesssim 1/2$, depending on u_i. A scenario to avoid a condensation by the ordinary gluon may require that the bulk mass parameter for all L-even fermions should be taken as $c \gtrsim 1/2$.

When fermions are KK modes, $g_{L,nmn}^{\text{brane}}$ are small for $y = 0$ and large for $y = L$ where $n = 1, 2, 3$. If u_i do not include suppression factors, the massless gluon strongly couples to KK modes of fermions at $y = L$. This property is almost independent of c.

Also in the case where KK-mode fermions are not identical levels, gauge couplings are non-vanishing. The couplings $g_{L,nm\ell}^{\text{brane}}$ divided by $(u_i/L)g_A/\sqrt{L}$, are shown in Fig. 7. where the fermions are L even and have different KK modes $n = 0, 1, 2, 3$ and $\ell = 0, 1, 2, 3$.

The c-dependence of the couplings $g_{L,0mn}^{\text{brane}}$ are similar to that of $g_{L,0m0}^{\text{brane}}$ in Fig. 6 where $m = 0, 1, 2, 3$ and $n = 1, 2, 3$. The c-dependence of the couplings $g_{L,0mn}^{\text{brane}}$ are similar to that of $g_{L,0m0}^{\text{brane}}$ in Fig. 6 where $m = 0, 1, 2, 3$ and $n = 1, 2, 3$.

Unlike the bulk couplings, the brane couplings have a small dependence of the KK level for the gauge boson. The summation of the KK mode can yield a large contribution.

Lastly the brane coupling g_m in Eq. (4.6) are given by the products of the boundary values $\left| N_m \chi_m \right|$ and z_i^{-3}. At $y = 0$, the values of $\left| \chi_m \right|$ are positive and negative alternately with respect to the KK level. In addition, the absolute value at each KK level is small. The coupling g_m for $y = 0$ seems small compared to other typical bulk and brane couplings. At $y = L$, the couplings are multiplied by z_L^{-3}. The coupling g_m also for $y = L$ seems small. Therefore bound states formed by brane fermions such as $\hat{\bar{Q}}\hat{U}$ tend to have small attractive forces compared to that of bulk fermions.

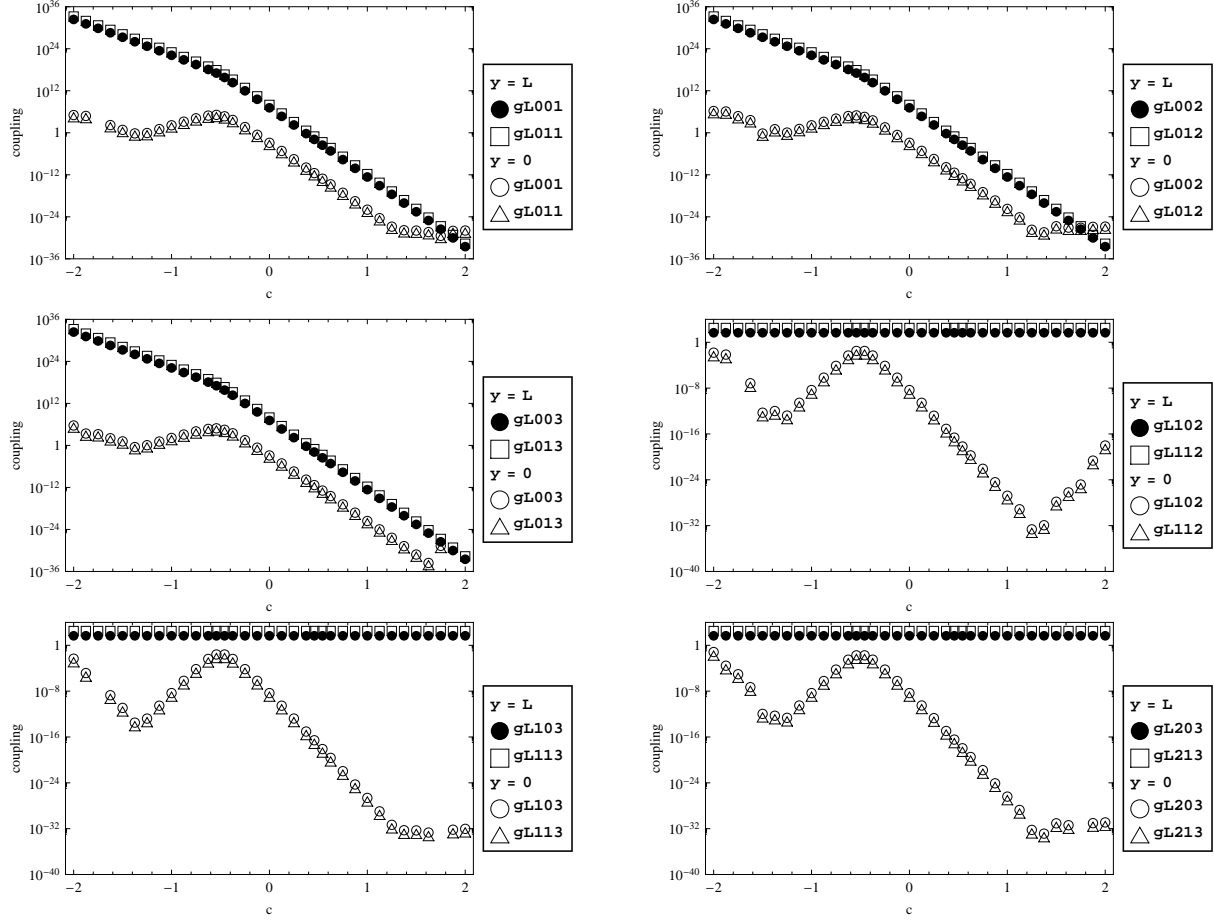It has been found that bulk and brane couplings notably differ in the dependences

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Figure 7: Coupling $g_{L,nm\ell}^{\text{brane}}$ divided by $(u_i/L)g_A/\sqrt{L}$, where the fermions are L even and have different KK modes $n = 0, 1, 2, 3$ and $\ell = 0, 1, 2, 3$.

on c and the KK level. The small bulk couplings for L-even fermions are summarized as $g_{L,n0\ell}^{\text{bulk}}$ for any c and $g_{L,0mn}^{\text{bulk}}$ for $c \lesssim -1/2$ where $n, \ell = 0, 1, 2, 3$ and $m = 1, 2, 3$. The small brane couplings for L-even fermions are summarized as $g_{L,0mn}^{\text{brane}}$ for $c \gtrsim 1/2$ where $n, m = 0, 1, 2, 3$ and $g_{L,nm\ell}^{\text{brane}}$ at $y = 0$ for $c \gtrsim 1/2$ where $n, \ell = 1, 2, 3$ and $m = 0, 1, 2, 3$. Here the convergence of brane couplings with respect to the summation of KK modes seems worse than that of bulk couplings. For R-even fermions, the discussion is parallel with $c \leftrightarrow -c$.

A scenario for a condensation to trigger electroweak symmetry breaking

We assume that zero-mode fermions and a zero-mode gauge boson are not condensed. The bulk coupling is $g_{000}^{\text{bulk}} = g_A/\sqrt{L}$ for any c. The brane couplings $g_{L,000}^{\text{brane}}$ and $g_{R,000}^{\text{brane}}$ depend on c. The mass parameter to satisfy this condition is $c \gtrsim 1/2$ for L-even fermions and $c \lesssim -1/2$ for R-even fermions.

For $c \gtrsim 1/2$, the coupling with a KK-mode gauge boson is large, $g_{L,010}^{\text{bulk}} \gtrsim 4g_A/\sqrt{L}$. Even zero-mode fermions receive strong attractive force through a KK-mode gauge boson. Also, even a zero-mode gauge boson can have large couplings with KK-mode fermions at

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$y = L$. When some of fermions and gauge bosons are zero modes and the others are KK modes, the couplings become strong.

A substantial point to realize electroweak symmetry breaking is that SU(3)-triplet S_{12} in Table 4 has a zero vacuum expectation value. A simple way to achieve this is to avoid a strong coupling for S_{12}. As the relative strength is 0.07, the bulk couplings whose maximum is about $6g_A/\sqrt{L}$ seem to remain small. The small brane couplings need at least $c \gtrsim 1/2$ for L-even fermions. From the viewpoint of both of the zero-mode coupling and the S_{12} coupling, the mass parameters $c \gtrsim 1/2$ for L-even fermions and $c \lesssim -1/2$ for R-even fermions are favored. This tendency is the same also for brane gauge couplings with one bulk fermion and one brane fermion due to the Ψ-$\hat{\Psi}$ mixing. Correspondingly, the contributions such as $\bar{\Psi}_{1L0}\Psi_{2R0}$ are localized at the brane with an intermediate-scale cutoff as described in Sec. 2.

When S_{12} does not have a vacuum expectation value, SU(2)-doublet scalars are only candidates for non-vanishing vacuum expectation values. The SU(2)-doublet scalars can be condensed through constituents with large attractive contributions from KK modes, while dynamical degrees of freedom for zero-mode fermions and zero-mode gluon are kept.

5 Boson masses and weak mixing angle

When electroweak symmetry breaking occurs, gauge boson and Higgs boson acquire their masses. We estimate how the gauge boson mass, weak mixing angle and Higgs boson mass are described in terms of the possible vacuum expectation values.

For simplicity, we focus on the composite Higgs $H_1(x, z)$ whose constituents are $\bar{Q}U(x, z)$. The extension for inclusion of H_2 and S_{13} is straightforward. Now the notation for a bulk composite Higgs is denoted as H. The inclusion of brane fields such as $\hat{H}_1$ will be shown explicitly.

The composite Higgs $H(x, z)$ has the interactions

$$\int d^4x dz \sqrt{\det(g_{KL})}\, g^{MN}(D_M H)^\dagger(D_N H)$$
$$+ \ a_i \int d^4x dz \sqrt{\det(g_{KL})}\, g^{\mu\nu}(D_\mu H)^\dagger(D_\nu H)kz\delta(z - z_i) \qquad (5.1)$$

where the overall factor has been normalized for the bulk kinetic term. For the present analysis, we regard a_i as parameters. The covariant derivatives includes the gauge couplings as $D_M H \supset (-ig_2 W_M - i(1/2)g_1 B_M)H$ where $Y_{\bar{Q}U} = 1/2$. The zero modes of gauge fields are given by $(1/\sqrt{L})W_\mu^{(0)}$ and $(1/\sqrt{L})B_\mu^{(0)}$.

When the condensation occurs, some energy would be released. Originally the fields Q_L and U_R have massless modes. The scalar bound state can fall on the minima of its potential. When the vacuum expectation value is generated as $\langle H \rangle = (0, v^{3/2})^T$, the light gauge fields have the mass terms as

$$\int d^4x\, \frac{v_{\text{eff}}^2}{2}\left[\frac{g_2^2}{L}\left(\frac{(W_\mu^{(0)1})^2 + (W_\mu^{(0)2})^2}{2}\right) + \frac{1}{2}\frac{g_1^2 + g_2^2}{L}\left(\frac{-g_2 W_\mu^{(0)3} + g_1 B_\mu^{(0)}}{\sqrt{g_1^2 + g_2^2}}\right)^2\right], \quad (5.2)$$

with $v_{\text{eff}}^2 = \left[(1 - z_L^{-2})/(2k) + a_0 + a_1 z_L^{-2}\right] v^3$. The weak mixing angle is given similarly to the standard model, $\cos\theta_W = m_W/m_Z = g_2/\sqrt{g_1^2 + g_2^2}$. It is independent of v_{eff}.

If the brane fields add the effect of a condensation corresponding to $\langle \hat{H} \rangle = (0, \hat{v})^T$, the effective expectation value v_{eff}^2 is replaced with $v_{\text{eff}+}^2 = v_{\text{eff}}^2 + \hat{v}^2$. The weak mixing angle is the same as the case of $\hat{v} = 0$.

The couplings $g_2/\sqrt{L}$ and $g_1/\sqrt{L}$ are effective four-dimensional couplings. In order that the gauge bosons have the masses at the weak scale, the v_{eff} should be of the order of $\mathcal{O}(100)$GeV. For $k = 4 \times 10^{12}$GeV and $a_0 \sim a_1 \sim L$, the definition of v_{eff} means $v^{3/2} \sim \mathcal{O}(10^7)$GeV.

From a four-dimensional standard representation for Higgs mass $\lambda v^2 = m_H^2$, the present Higgs mass is estimated as $\lambda v^3 = m_H^2$ where λ is the coefficient for a fermion four-point interaction. When strongly-coupled effects for gauge interactions are denoted as N_{np}, the coupling is given by $\lambda \sim g^2 N_{np} \sim g_4^2 L N_{np} \sim g_4^2 (\log z_L/k) N_{np}$. Then the Higgs mass squared is given by $m_H^2 = g_4^2 \log z_L \cdot N_{np} v^3/k \sim g_4^2 N_{np} \cdot \mathcal{O}(1000)$GeV. In the present setup, the KK fields are heavy. As shown in Table 1, the lightest mass is about 10 TeV. In order that the tree level unitarity is not violated, the Higgs boson may need to contribute below the scale where KK fields become dynamical. If N_{np} is not extremely large, the renormalization group running from the scale $v^{3/2}$ to the weak scale may drive the Higgs boson mass to $\mathcal{O}(100)$GeV.

6   Conclusion

We have systematically examined gauge couplings for fermions in a warped space. Here bulk and brane coupling of bulk fermions and brane coupling of brane fermions have been calculated. This has been related to binding strengths in the most-attractive-channel approximation.

From the quantum number given, there are six color-triplet scalars for one-generation bulk fermions. For the three scalars S_7, S_8, S_9, one of the constituents $\bar{\Psi}_1$ and Ψ_2 is necessarily a KK mode. The two scalars, S_{10} and S_{11} do not receive attractive force. Then the dangerous color-triplet scalar is only S_{12}. Brane fermions yields the correspondent $\hat{S}_{12}$. The scenario is that these bound states are prevented from having their vacuum expectation values while weak-doublet scalar bound states have non-vanishing vacuum expectation values to trigger symmetry breaking.

The boundary values of the mode functions significantly depend on the bulk mass parameters for zero-mode fermions. The c-dependence of KK-mode fermions and both modes of gauge bosons is small. Therefore, the brane couplings for zero-mode fermions most strongly depend on c. To avoid large couplings of zero-mode fermions such as a zero-mode gluon-fermion-fermion yields constraints on c. The bulk and brane gauge couplings of bulk fermions differ in the convergence of the KK summation. From this point of view, the brane gauge couplings are worse than the bulk couplings. The contributions of the brane gauge coupling at each KK level can be small depending on the bulk mass parameter. The brane gauge couplings of brane fermions tend to be smaller than the brane gauge couplings of bulk fermions. The bulk coupling for zero-mode gluon and fermions is given by $g_{000}^{\text{bulk}} = g_A/\sqrt{L}$ for any c. The condition that brane couplings $g_{L,000}^{\text{brane}}$ and $g_{R,000}^{\text{brane}}$ are at most $\sim g_A/\sqrt{L}$ gives $c \lesssim 1/2$ for L-even fermions and $c \gtrsim -1/2$ for R-even fermions.

The condition that the couplings for the color-triplets are at most $\sim g_A/\sqrt{L}$ requires the same region for c.

We have found an interesting property for the bulk couplings. For $\bar{\tilde{\Psi}}_n \gamma \cdot A_m \tilde{\Psi}_\ell$, large couplings are given for $\ell - n + 1 = m$ in Eq. (4.7). It is curious that this relation is not to represent the conservation of KK modes. The well-known $g^{\text{bulk}}_{L,010} > g^{\text{bulk}}_{000}$ in a wide region of c is characteristic. In particular, for $c \gtrsim 1/2$ the KK gluon coupling is clearly large $g^{\text{bulk}}_{L,010} \gtrsim 5g^{\text{bulk}}_{000}$. Large couplings of KK gluon are a necessary consequence when the strong coupling for a zero-mode gluon is avoided.

The gauge boson masses, weak mixing angle and Higgs boson masses have been related to the vacuum expectation values. The weak mixing angle is written in terms of the coupling constants as in the standard model. For $k = 4 \times 10^{12}\text{GeV}$ and $a_0 \sim a_1 \sim L$, the vacuum expectation value has been estimated as $v^{3/2} \sim \mathcal{O}(10^7)\text{GeV}$. As the KK fields are heavy and the lightest mass is about 10TeV, the Higgs boson may need to be lighter than them so that unitarity is not violated. The renormalization group flow from the high scale $v^{3/2}$ to the weak scale may make the Higgs boson mass light.

We have examined the aspect of the gauge couplings for the self-breaking of the standard model gauge symmetry in the warped space. Many issues such as the actual vacuum expectation values and the flavor mixing need to be examined in more detail. This and the comparison with experimental data would require some fundamental development of estimation for quantum corrections with extra dimensions.

Acknowledgments

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